Complications of cylindrical anisotropy on the properties of fibres

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Cylindrically orthotropic materials having linear elastic stress–strain relations can possess a unique type of coupling between the radial and tangential directions. This type of anisotropy occurs when fibres have onion skin or radial morphologies which are characteristic of some graphite fibres and poly-*p*-phenylene terephthalamide fibres. Pulling on a long slender fibre having this type of anisotropy does not produce a uniaxial state of stress because the unequal Poisson ratios also cause both radial and tangential stresses. The magnitude of these predicted transverse stresses in relation to the axial stress is similar to the relationship of strengths in these principal directions. Hence, it is conceivable that fibre failure could be caused by fibre splitting from high transverse stresses rather than the longitudinal tensile stresses. Temperature changes and other factors causing dimensional changes such as swelling and pressure can similarly cause large stresses if the radial and tangential expansion coefficients are different. This type of axial–transverse coupling is known to limit certain designs in composite structures having this type of anisotropy, yet little has been done to understand fibres possessing these characteristics. This paper examines the complex state of stress that can result in fibres having cylindrically orthotropic linear elastic properties for several simple modes of deformation.

(Keywords: cylindrical anisotropy; fibres; stress–strain relations)

INTRODUCTION

The high degree of axial molecular alignment observed for many polymeric high performance fibres would indicate strong mechanical anisotropy exists in these fibres. Mechanical properties in the fibre direction, such as modulus and strength, are high[1] because they derive from the oriented polymer backbone, whereas transverse and shear properties, which are dependent upon the much weaker secondary forces between chains, are considerably lower in value than the axial properties. Additionally, depending on the morphological development of the fibre structure, the possibility exists for anisotropy within the fibre cross section such that the radial and hoop directions are not mechanically equivalent.

The clearest examples of anisotropic bodies are certainly the single crystals of various materials. In practical situations, however, most objects are composed of random arrangements of anisotropic domains giving rise to overall isotropic behaviour. On the other hand, macroscopic anisotropy is observed when such random arrangements give way to ordering by various types of growth or processing conditions. These higher degrees of ordering may be based at the molecular level or on a larger scale such as is the case for wood and composite materials. An example of anisotropic fibre structure is found in carbon and graphitic fibres where either a radial or onion skin morphology[2] may be developed. In this case, one transverse principal direction (either radial or hoop) is parallel to the graphitic basal planes whereas the other principal direction is perpendicular to the basal planes and hence the cross section exhibits anisotropy similar to that of single crystal graphite[3,4]. Transverse fibre anisotropy also exists for poly-*p*-phenylene terephthalamide (PPTA) fibres because of the radial arrangement of hydrogen bonded sheets of the PPTA molecules[5,6]. Haraguchi *et al.*[7] have also noted the development of preferred orientation of PPTA for solutions coagulated in various nonsolvents. Such preferred orientations could also possibly develop in other polymeric fibres due to various processing conditions and hence lead to cylindrically anisotropic structures.

While morphological anisotropy of various fibres is known, little attention has been given to the examination of anisotropic fibre behaviour. The rather small lateral dimensions of available high performance fibres (10–20 microns), along with their cylindrical geometry pose a formidable challenge to the experimentalist desiring knowledge other than of their axial mechanical properties. While certain high performance materials are amenable to study in other geometries of larger dimensions, most high performance polymeric materials are generally only available in fibre form, restricting the available means of examining their anisotropy. In this work, a general consideration of cylindrically orthotropic fibre behaviour is presented to explore the possible states of stress for simple test geometries. It will be shown that fascinating mechanical response occurs when the properties in the plane of the fibre cross section are not isotropic. In fact, simple states of stress are almost impossible for such anisotropic fibres. These effects are not well known and can be of considerable importance to those attempting to understand the properties of these materials. It is the intent of this work to draw attention to possible implications arising from cylindrical anisotropy to promote interest in this area.

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GENERAL CONSIDERATIONS OF ANISOTROPY

The texts of S. G. Lekhnitskii[8,9] are among the best sources of information on the stress analysis of anisotropic materials, especially concerning cylindrical anisotropy. We first review relevant aspects of mechanical anisotropy which, for the most part, is extracted from Lekhnitskii's work.

The mechanical properties of an anisotropic elastic solid which can be treated as a continuous medium are defined by the generalized Hooke's Law

$$\varepsilon_i = a_{ij}\sigma_j \qquad (i, j = 1 \cdots 6) \tag{1}$$

for small strains where a linear relationship between stresses and strains is assumed. In equation (1) a summation over j is implied where a_{ij} represents the compliance constants, and σ_j and ε_i are the respective stress and strain measures using the generally employed contracted engineering notation[8–11] (we also adopt the notations τ and γ for shearing stresses and strains). A consideration of the required symmetries of the stress and strain measures and of the conservation of energy, gives the number of independent components of a_{ij} as 21 (refs 8–11). Further reduction in the number of independent components of a_{ij} can only be made through restrictions imposed by the symmetry properties of the material. While it is certainly possible to deal with such a general anisotropic material it seems reasonable for the present purpose to consider only a few particular cases, where certain symmetry restrictions may be imposed. In this light, the most general case to be considered is that of a fibre possessing cylindrical orthotropy.

For a cylindrically orthotropic material it is assumed that the fibre possesses symmetry with respect to three mutually orthogonal planes (r, θ, z), which reduces the number of independent compliance constants from 21 to 9. The resulting compliance coefficients may then be expressed in matrix form as:

$$a_{ij} = \begin{vmatrix} a_{11} & a_{12} & a_{13} & 0 & 0 & 0 \\ a_{12} & a_{22} & a_{23} & 0 & 0 & 0 \\ a_{13} & a_{23} & a_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & a_{66} \end{vmatrix} \tag{2}$$

Before considering particular implications of orthotropic anisotropy it is useful to consider one further case of symmetry, that of transverse isotropy which is more commonly referred to as fibre symmetry. In this case no distinction between radial and hoop directions is made, that is to say that all planes perpendicular to the fibre axis are treated as being isotropic. The number of independent compliance coefficients is then further reduced from nine to five where equation (2) is simplified by the relations, $a_{11} = a_{22}$, $a_{13} = a_{23}$, $a_{44} = a_{55}$ and $a_{66} = 2(a_{11} - a_{12})$ for the transversely isotropic fibre.

CYLINDRICAL ORTHOTROPY

Axial tension/compression

One of the simplest applications to consider is that of determining the state of stress of a fibre in equilibrium under the action of a tensile force, P, directed along the fibre axis, z. Lekhnitskii[8,9] has shown that in this case, for a cylindrically orthotropic rod of radius b the distribution of stresses in the rod is given by:

$$\begin{aligned} &\tau_{r\theta} = \tau_{rz} = \tau_{\theta z} = 0 \\ &\sigma_r = (Ph/T_0)[1 - (r/b)^{k-1}] \\ &\sigma_\theta = (Ph/T_0)[1 - k(r/b)^{k-1}] \\ &\sigma_z = (P/T_0) - (Ph/T_0 a_{33})[a_{13} + a_{23} - (a_{13} + k a_{23})(r/b)^{k-1}] \end{aligned} \tag{3}$$

where

$$T_0 = \pi b^2 + \pi b^2 \frac{h}{a_{33}} \frac{k-1}{k+1} (a_{23} - a_{13}) \tag{4}$$

and h and k are compound constants related to the compliance coefficients by

$$\begin{aligned} h &= \frac{a_{23} - a_{13}}{a_{11} - a_{22} + (a_{23}^2 - a_{13}^2)/a_{33}} \\ k &= \sqrt{\frac{a_{11}a_{33} - a_{13}^2}{a_{22}a_{33} - a_{23}^2}} \end{aligned} \tag{5}$$

The set of equations (3) illustrates that for a cylindrically orthotropic fibre subjected to a tensile load, P, not only is the axial stress σ_z dependent on radial position but that radial and hoop stresses (also radially dependent) are present. The relative magnitudes of the hoop and radial stresses are sensitive to differences in the compliance constants between the radial and hoop directions, most notably the difference $(a_{23} - a_{13})$ expressed in the parameter h. This difference for a transversely isotropic fibre is equal to zero, where no distinction between radial and hoop directions exists. For the case of a transversely isotropic fibre the system of equations (3) reduces to

$$\begin{aligned} &\sigma_z = P/\pi b^2 \\ &\sigma_r = \sigma_\theta = \tau_{r\theta} = \tau_{rz} = \tau_{\theta z} = 0 \end{aligned} \tag{6}$$

equivalent to the elementary distribution of stresses for an isotropic fibre.

The relative magnitudes of any radial or hoop stresses which may develop in a cylindrically anisotropic fibre supporting a tensile load P may have important consequences regarding failure of the fibre when anisotropic failure characteristics are taken into account. If, for example, transverse strengths are much lower than axial strength, it is conceivable that a fibre tensile failure could be the result of a transverse failure rather than a true axial failure. It is also interesting to note that the sign of the radial and hoop stresses is dependent upon the direction of the axial load P (tensile or compressive), as well as on differences in the compliance constants (see equations 3). Therefore if axial tension were to produce compressive transverse stresses, axial compression would produce transverse tension, or vice versa, creating the possibility of differences in tensile and compressive failure stresses if such failures are related to transverse failures.

Carbon fibres. It is instructive to examine the relative magnitudes of the transverse stresses produced in a cylindrically anisotropic fibre by examining a few model fibre structures. As previously mentioned, it is possible for graphite fibres to exhibit either an onion skin or a radial arrangement of the graphitic basal planes[2]. To estimate the state of stress in a model graphite fibre the

Table 1 Compliance coefficients for model graphite fibres. Data from reference 10

a_{ij}	Radial (GPa^{-1})	Onion skin (GPa^{-1})
a_{11}	0.00098	0.0275
a_{22}	0.0275	0.00098
a_{33}	0.00098	0.00098
a_{13}	−0.00016	−0.00033
a_{23}	−0.00033	−0.00016

compliance coefficients for single crystal graphite[12] have been used for the analysis of a perfect radial and a perfect onion skin morphology. *Table 1* lists the relevant compliance coefficients (a_{ij}) for the two cases. The radial dependence of the axial stress σ_z will be minor and is not of importance for the present discussion.

Radial (σ_r) and hoop (σ_θ) stresses produced in these model graphite fibres have been calculated based on equations (3) and the compliance constants of *Table 1*. *Figure 1* summarizes the predicted values of these stresses as a function of radial position (r/b) within the fibre cross-section. The values of σ_r and σ_θ have been normalized by the 'isotropic' value of axial stress ($\sigma_z = P/\pi b^2$) and are presented as percentages of the axial stress level. Both hoop and radial stresses for the onion skin morphology are bounded within the cross section whereas the ideal radial morphology exhibits stress singularities at the fibre centre. For axial tension, the onion skin morphology is characterized by tensile hoop and radial stresses at the fibre centre having a value of 0.64% of the applied axial stress. At the fibre surface the radial stress is zero while the hoop stress is compressive of relative magnitude 2.8%.

The radial morphology, however, exhibits a tensile hoop stress at the fibre surface of $0.0052\sigma_z$ with zero radial stress while both hoop and radial stresses at the fibre centre approach infinite compression when the applied axial stress is tensile. Infinite core stresses can, however, be excluded from the present consideration simply by replacing this region by a small isotropic cylinder or by having a void at the centreline, an example of which will be given later. (These stress singularities will not be fully dealt with in this discussion, see references 8 and 9 for a further discussion.)

The relative importance of any hoop or radial stresses which may develop in an orthotropic fibre may be considered in light of the strength characteristics in the principal directions. If the magnitudes of these stresses are such that the radial or hoop strengths are not approached then only minor significance is given to their existence (except of course for the precise determination of compliance coefficients). If, on the other hand, these stresses approach the strength values in the radial or hoop directions, a closer examination of fibre failure is warranted. Considering the idealized form of the model graphite fibres presented here, a comparison of theoretical strengths seems justified. It is well known[13–15] that the theoretical tensile strength relative to modulus (σ_b/E) is on the order of 0.1, so that characteristic strengths may be approximated from knowledge of the compliance constants given in *Table 1*. The relevant strengths for graphite are those parallel and perpendicular to the basal planes, which from *Table 1* are estimated as 102 GPa and 3.6 GPa respectively. It should be mentioned that the observed strengths of materials are generally an order of magnitude lower (i.e. $0.01E$) than such theoretical values (flaws, dislocations, etc.) but because only ratios of these strengths are important for the present discussion the comparison of theoretical values seems warranted.

The ratio of perpendicular to parallel basal plane strengths for these idealized fibres is equal to 0.036 based on the above considerations. The direction perpendicular to the basal planes would be radial for the onion skin structure and hoop for the radial structure. From *Figure 1* it is observed that the maximum tensile stress perpendicular to the graphite sheets is only 0.5%–0.6% of the axial stress for the two structures. This relative value is below the 3.6% stress level predicted for transverse failure to become probable. It is therefore unlikely that the presence of cylindrical anisotropy in graphite fibres is a dominant factor determining tensile failures especially in light of the number of idealizations employed in the analysis. However, as mentioned earlier, the sign of the transverse stresses changes in going from axial tension to axial compression where for the radial model quite large transverse tensions would be present near the fibre core.

It is interesting to note for the idealized models discussed above that the mechanical behaviour of the graphite fibres is not strictly that of an orthotropic body possessing three mutually perpendicular planes of symmetry each with unique elastic constants. The elastic symmetry is more simplified than the general orthotropic case by having considered the graphitic sheets as planes

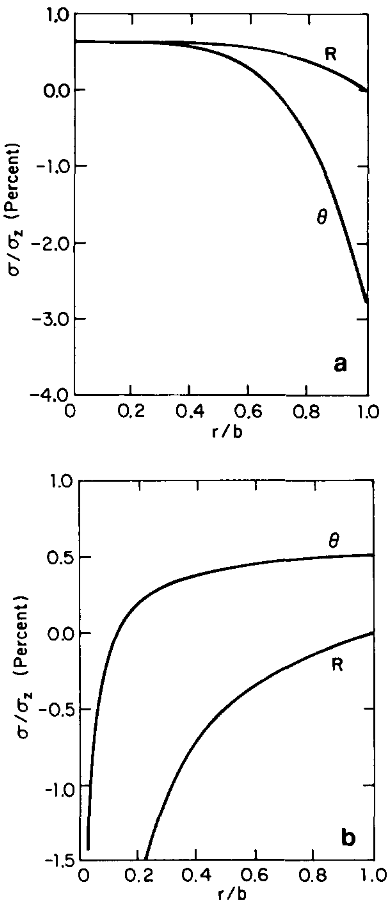

Figure 1 (a) Radial (R) and hoop (θ) stress distributions in a model onion skin graphite fibre under axial tension (σ_z). (b) Model radial graphite fibre

Table 2 Compliance coefficients for model PPTA fibres

a_{ij}	Case I (GPa^{-1})	Case II (GPa^{-1})
a_{11}	0.0625	0.0625
a_{22}	1.316	0.112
a_{33}	0.00769	0.00769
a_{13}	−0.000625	−0.000625
a_{23}	−0.01316	−0.00112
h	0.01	0.01
k	0.22	0.75
$T_0/\pi b^2$	1.01	1.00

of isotropy possessing identical elastic constants in two of the three directions.

PPTA fibres. As a further example of cylindrical orthotropy an idealized PPTA fibre is considered where structural evidence[5,6] suggests a radial arrangement of hydrogen bonded planes of the poly-(*p*-phenylene terephthalamide) molecules. In this case the fibre properties in the axial, radial and hoop directions would be related to covalent chemical bonding, hydrogen bonding and van der Waals bonding, respectively, representing a more general orthotropic structure than for graphite.

While the distributions of radial and hoop stresses for a radial PPTA fibre are expected to be of a similar form as for radial graphite (*Figure 1*), the magnitudes of these stresses are dependent upon the specific compliance coefficients of the structure. Exact values of all of the compliance coefficients needed to solve for the stress distributions are unavailable but from those that are known and from comparisons with other anisotropic materials a working set of constants can be obtained for our model fibre. The axial modulus of PPTA fibres is approximately 130 GPa (refs 16–19), yielding a compliance a_{33} of 0.00769 GPa^{-1}. Values of the transverse moduli or compliances are not well known and vary depending on their method of measurement or estimation. Northolt and van Aartsen[20] estimate a modulus in the hydrogen bonding direction of 16–29 GPa based on hydrogen bonding force constants taken from the infrared spectroscopic literature. Phoenix and Skelton[17] have obtained an average transverse modulus of 760 MPa based on transverse compression experiments of Kevlar® 49 [PPTA] filaments assuming transverse fibre isotropy. Uniaxial fibre composite data[16] suggest a transverse modulus of 8.9 GPa based on a simple series analysis of the composite transverse modulus.

Using the lower value for the hydrogen bonding modulus of 16 GPa the radial compliance of an idealized PPTA fibre may be taken as $a_{11}=0.0625\ GPa^{-1}$. Compliance values of 1.316 GPa^{-1} and 0.112 GPa^{-1} for a_{22} are obtained from the transverse compression and composite data, respectively. The remaining two constants a_{13} and a_{23} must be estimated by comparison to other materials due to the lack of experimental data. By comparison with graphite[12], pine wood[8] and cellulose[21], for example, the ratio of a_{13}/a_{11} or a_{23}/a_{22} is expected to be on the order of −0.01 to −0.03. As a first approximation a value of −0.01 will be employed for these ratios for the analysis of hoop and radial stresses in an axially loaded idealized PPTA fibre (hydrogen bonded planes radially arranged). By using the same value of this ratio (a_{13}/a_{11} or a_{23}/a_{22}) to obtain both coefficients a_{13} and a_{23} it is tacitly assumed that the radial and hoop Poisson effect are the same, a condition which will be relaxed later.

Table 2 lists the compliance parameters used to evaluate radial and hoop stresses in the idealized radial PPTA fibre. *Figures 2* and *3* summarize the results in the same form as for the graphite fibres. The weak direction for this fibre is expected to be the hoop direction which is perpendicular to both the chain and hydrogen bonding directions so that attention will be focused on the values of hoop stress σ_θ. For these radial structures the hoop stress is found to be tensile except near the fibre centre where, as seen before, the equations predict infinite compression at the centreline for an axially applied tensile load. The maximum value of hoop stress occurs at the fibre surface ($r=b$) where values of 0.77% and 0.25% of the axial stress are calculated for cases I and II (see *Table 2*), respectively. If the Poisson constraint (−0.01) is relaxed for case II such that a_{23} assumes the value $-0.00336\ GPa^{-1}$ (i.e. $a_{23}/a_{22}=-0.03$, $a_{13}/a_{11}=-0.01$), the magnitudes of the stresses σ_r and σ_θ are increased by a factor of 5.68. In this case the maximum value of σ_θ is 1.52% of the axial stress. Other variations of the compliance coefficients yield similar results within the range of predictions obtained from these examples.

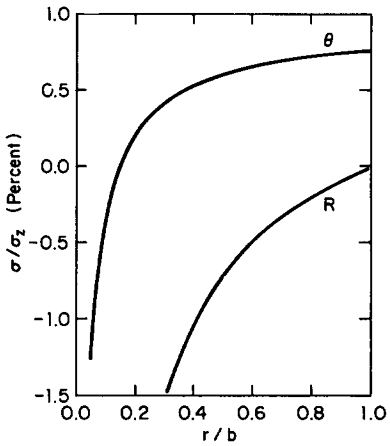

Figure 2 Radial (R) and hoop (θ) stress distributions in a model (Case I) radial PPTA fibre under axial tension (σ_z)

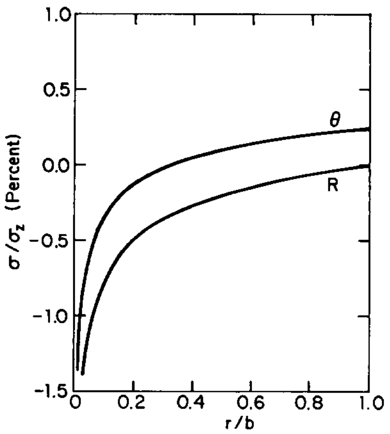

Figure 3 Radial (R) and hoop (θ) stress distributions in a model (Case II) radial PPTA fibre under axial tension (σ_z)

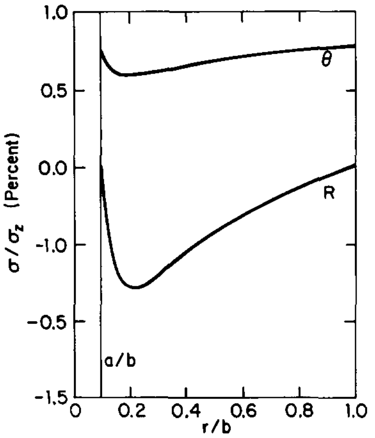

Figure 4 Radial (R) and hoop (θ) stress distributions in a hollow model (Case I) radial PPTA fibre under axial tension (σ_z)

The relative importance of transverse stresses are of course determined by the characteristic strengths in the transverse directions relative to the longitudinal strength. Based on the theoretical strength relative to modulus, the hoop strength is very approximately 0.6–7% of the axial strength based on the compliances used in *Table 2*. Reference 16 gives a transverse strength of 29.6 MPa for a uniaxial Kevlar® 49 (PPTA) composite which, if the failure was associated with a transverse fibre failure, would correspond to the transverse fibre strength. This value of transverse strength is 0.8% of the axial strength based on data from the same reference. Recalling that the maximum ratios of hoop to tensile stresses predicted for the models is in the range of 0.25%–1.42%, it seems probable that the transverse stresses could be an important consideration in the tensile deformation of an ideal cylindrically anisotropic PPTA fibre. It is perhaps even more important with regard to the compression behaviour.

Figure 4 illustrates the stress distributions predicted for a Case I PPTA fibre having a small hole along the fibre centreline. Here again the analysis has been taken from the works of Lekhnitskii[8,9]. The presence of the axial hole (hollow fibre) removes the complication of having stress singularities at the fibre centre which were found for the solid fibres (see *Figures 1–3*). The relative magnitudes of the predicted radial and hoop stress are comparable to those discussed above for such a model fibre under axial tension or compression.

Other deformation modes

Stress analysis has also been performed for these model cylindrically orthotropic fibres in bending and torsion and the detailed results are presented elsewhere[22]. The elastic constraint of cylindrical orthotropy simplifies the analysis of generalized torsion to that of ordinary torsion and the distribution of stresses is obtained in the same way as for an isotropic fibre. In bending, however, both radial and hoop stresses are predicted for cylindrically orthotropic fibres. The maxima of the radial and hoop stresses in bending will occur along the direction perpendicular to the neutral axis, as will the axial stress.

SUMMARY

Considerable morphological evidence exists indicating that both carbon (graphite) fibres and PPTA (Kevlar®) fibres can exhibit preferential ordering within the fibre cross section (i.e. a layered radial or onion skin structure). While such investigations of morphological order have revealed anisotropic fibre structures, little attention has been given to the mechanical anisotropy expected of such structures. Anisotropic elasticity theory was applied to the examination of the state of stress of model radial and onion skin fibre morphologies for some simple mechanical test geometries to draw attention to the possible complications arising from mechanical anisotropy. For both bending and axial loading, radial and hoop stresses are predicted in addition to the axial stresses for cylindrically orthotropic fibres as opposed to a transversely isotropic fibre where no radial or hoop stresses are predicted.

One aspect of the importance of radial and hoop stresses in cylindrically orthotropic fibres experiencing axial tension/compression or bending is determined by the relative strength characteristics in these principal directions. For the model radial PPTA fibre considered, the magnitude of the predicted hoop stresses relative to axial stress is of the same order of magnitude of estimated and experimentally measured transverse strength relative to axial strength values. These results suggest that in light of directional strength dependences, transverse stresses arising due to material orthotropy should be considered in determining strength characteristics of such anisotropic fibres. An exact evaluation of the importance of possible radial and hoop stresses would require a detailed knowledge of compliance coefficients as well as of the true fibre structure, including factors such as skin–core morphology. Additionally, transverse stresses may arise due to anisotropy of thermal expansion coefficients as well as of swelling coefficients[23]. The presence of such transverse stresses also dictates the necessity of a more thorough analysis to precisely determine even the axial elastic constants as shown by the complexity of the axial stress distributions revealed in equations (3). At present, however, the degree to which cylindrical anisotropy influences the interpretation of fibre performance is unknown because the engineering constants for such materials have never been determined in this context. Further experimental and theoretical work is encouraged.

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